

Universal Transverse Conductance between Quantum Hall Regions and $(2 + 1)$ D Bosonization

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Abstract

Using bosonization techniques for $(2 + 1)$ D systems, we show that the transverse conductance for a system with general current interactions, when measured between perfect Hall regions is not renormalized at low temperatures. Our method extends two results we have recently obtained on low dimensional fermionic systems: on the one hand, the relationship between universality of Landauer conductance and universality of bosonization rules for $(1 + 1)$ D systems, and on the other hand, the universal character of the bosonized topological current associated to a $(2 + 1)$ D fermionic system with

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I. INTRODUCTION

Universal properties of interacting fermionic systems have always attracted the attention of physicists. The reason is that a clean and exact behavior for a system containing impurities and complicated particle interactions must be associated to a strong constraint imposed by a simple physical principle. Low dimensional condensed matter systems offer a variety of situations where these universal phenomena occur.

For 1D systems such as quantum wires, recent experiments¹ showed that, at low temperatures, the measured Landauer conductance is equal to the quantum e^2/h for each propagating channel. Soon it was understood that, for a *finite* Luttinger liquid wire, the conductance is not renormalized by the interaction in the wire²⁻⁴, since it is dominated by the noninteracting electron gas in the leads. Moreover, a general relationship between universality in transport properties of 1D systems and chiral symmetry has been recently proposed^{5,6}.

In Ref. 6, following Maslov and Stone's reasoning², one of us showed that perfect conductance should also occur, at low temperatures, for a 1D incommensurate charge density wave (CDW) system adiabatically connected to Fermi liquid leads. This result agrees with that obtained in Ref. 7, where transport of charge in disordered mesoscopic CDW heterostructures was studied within the Keldysh formalism. In contrast to the elaborate calculation of Ref. 7, we presented very simple physical arguments based on the existence of a chiral (anomalous) symmetry for the system as a whole (CDW plus leads) when the phase of the CDW order parameter is dynamic.

In Ref. 6, we also stressed the important role played by the finiteness of the system, the adiabatic contacts to the reservoirs, and the universal character of the bosonization rules, showing that a general 1D structure of the Fermi-liquid/finite-system/Fermi-liquid type displays a perfect Landauer conductance at low temperatures, provided the finite system presents an (anomalous) chiral symmetry. The adiabaticity allows the extension of the symmetry to the system as a whole in such a way that an anomalous chiral current is always

present when a bias voltage is applied; outside the sample, this current is associated to the transport of free fermions.

These general properties cause the charge transport through the system to be dominated by the reservoirs, i.e., chiral symmetry is the physical principle behind the universality of Landauer conductance in 1D systems. The natural language we used to study these systems is bosonization, which maps the initial $(1 + 1)$ D fermionic system into a bosonic one, describing collective excitations represented by a scalar field^{8–11}.

In connection with 2D systems, the understanding of the impurity independence of the transverse conductance in the quantum Hall effect was initiated by Prange¹², but it was after the work by Laughlin¹³ and Halperin¹⁴ that the underlying mechanism for the universal character of the transverse conductance was associated to the principle of gauge invariance. This is the accepted explanation for the amazing degree of accuracy for the transverse conductance, which is insensitive to such details as the sample's geometry and the amount of impurities. A very interesting topological interpretation of this fact can be found in Ref. 15.

The aim of this work is to show that bosonization is also a natural language to describe universal transport properties for 2D systems.

The generalization of the bosonization technique to higher dimensions is recent. In the context of condensed matter systems, the first attempts to bosonize a Fermi-liquid in higher dimensions were presented in Refs. 16 and 17. In Ref. 18 the shape fluctuations of the Fermi surface were studied in a very systematic way, and a detailed analysis of the Landau theory as a fixed point of the renormalization group were presented.

In the context of quantum field theory, an important activity on bosonization in higher dimensions was initiated in the beginning of the nineties,^{19–26}. In particular, the bosonization of a massive $(2 + 1)$ D Dirac field is achieved in terms of a gauge theory, where the Chern-Simons action plays a fundamental role, and the fermionic current is mapped into

the topological current $\epsilon^{\mu\nu\rho}\partial_\nu A_\rho$. This bosonization, in contrast to the abovementioned Fermi-liquid case, deals with parity breaking systems (in $(2+1)$ dimensions, the mass term $M\bar{\psi}\psi$ breaks parity). In this paper, this is the kind of systems we will be interested in, namely, 2D systems displaying the following properties:

- gauge invariance (charge conservation).
- a gap in the low lying charged excitations.
- Lorentz or Galilean invariance.
- Parity or time reversal symmetry breaking.

Besides studying the $(2+1)$ D fermionic case, which is the simplest one, we will also study the physically relevant nonrelativistic case, where 2D spinless fermions are submitted to an external magnetic field B ; here the gaps are provided by the Landau quantization.

It is worthwhile stressing here that the two examples we will consider have a quite different underlying physics. One of the consequences is that the particular values for the universal transport properties of these systems will be different; however, in both cases, the proof of universality will be similar as it relies on the general properties shared by them. The role played by bosonization is to implement all these properties in a very simple and compact way.

Essentially, we will extend two results we have recently obtained for low dimensional fermionic systems. One of them is the relationship between the universality of Landauer conductance and the universality of the mapping between the fermionic current and the bosonized topological current $\epsilon^{\mu\nu}\partial_\nu\phi$, in 1D systems⁶. The other one, is the universal character of the bosonized topological current $\epsilon^{\mu\nu\rho}\partial_\nu A_\rho$, for a class of $(2+1)$ D systems²⁴; this will be related to the universality of the transverse conductance, for a system with general current interactions, when measured between “perfect Hall regions” (where the parity breaking parameter, M or B , goes to infinity). As a byproduct, we shall also see that the Aharonov

and Casher results²⁷ for fermionic systems with spin, will remain valid when interactions are included.

Thus, we will see that bosonization is a method that unifies universal physical behaviors associated to systems with different dimensionality. While, in 1D, bosonization is a simple way to display the anomalous properties of chiral symmetry, in 2D, it is appropriate to display the gauge invariance of the effective fermionic action, that is, the physical principle behind the universal character of transverse conductance. Then, it is no by chance that, in this framework, we shall be able to derive the universal transverse transport.

Although a closed expression for the bosonized action in higher dimensions is still lacking, the gauge and topological structure of the bosonized theory, and the universal character of the bosonized currents are the only properties we shall need to derive our results. In analogy with the 1D case, where the Fermi-liquid in the reservoirs imposes strong constraints on the Landauer conductance, we shall see that perfect Hall regions will impose strong constraints on the transverse conductance. As before, an adiabatic transition between the interacting and the noninteracting regions will be needed. From a physical point of view, this condition corresponds to nondissipative contacts.

This paper is organized as follows: In section II we review and compare the functional bosonization technique in $(1+1)$ D and $(2+1)$ D. In §III we briefly review the relationship between the universality of the bosonization rules in $(1+1)$ D and the universality of Landauer conductance in 1D finite systems. Section IV has the main results of this paper where we deduce the universal transverse conductance between “perfect Hall regions”, using the universal mappings between currents. In §V we extend our results to the nonrelativistic case, obtaining the universal properties of transverse currents, in the integer as well as in the fractional quantum Hall effect. Finally in §VI we discuss our results and give our conclusions.

II. THE BOSONIZATION TECHNIQUE

In order to deal with the general bosonization structure in higher dimensions, it is convenient to follow the path-integral approach of Refs. 20,21 and 24. The free fermionic partition function is

$$Z_0[s] = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{iK_F[\psi] - i \int d^\nu x j^\mu s_\mu}, \quad (1)$$

where K_F is a free fermionic action term and $j^\mu = \bar{\psi} \gamma^\mu \psi$ (ν is the dimensionality of space-time).

Using gauge invariance (for $\nu = 2$ we suppose a gauge invariant regularization), we have $Z_0[s] = Z_0[s + b]$, where b is a pure gauge field. Then, up to a global normalization factor, we can write

$$Z_0[s] = \int \mathcal{D}b|_{\text{pure gauge}} Z_0[s + b]. \quad (2)$$

This functional integration can also be carried over the whole set of gauge fields b by imposing an appropriate constraint. For $\nu = 2$, the constraint is given by $\delta[\epsilon^{\mu\nu} \partial_\mu b_\nu]$. Then, exponentiating the delta functional by means of a scalar lagrange multiplier $\phi(x)$ and shifting $b \rightarrow b - s$, the bosonized representation is obtained,

$$Z_0[s] = \int \mathcal{D}\phi e^{iK_B[\phi] - i \int d^2 x s_\mu \epsilon^{\mu\nu} \partial_\nu \phi}, \quad (3)$$

where

$$e^{iK_B[\phi]} = \int \mathcal{D}b Z_0[b] e^{i \int d^2 x b_\mu \epsilon^{\mu\nu} \partial_\nu \phi}. \quad (4)$$

For $\nu = 3$, the constraint is $\delta[\epsilon^{\mu\nu\rho} \partial_\nu b_\rho]$ and the delta functional is exponentiated by means of a vector field Lagrange multiplier A_μ . Following the same steps as before, we obtain

$$Z_0[s] = \int \mathcal{D}A e^{iK_B[A] - i \int d^3 x s_\mu \epsilon^{\mu\nu\rho} \partial_\nu A_\rho}, \quad (5)$$

where

$$e^{iK_B[A]} = \int \mathcal{D}b Z_0[b] e^{i \int d^3x b_\mu \epsilon^{\mu\nu\rho} \partial_\nu A_\rho}. \quad (6)$$

Differentiating $Z_0[s]$, we read from Eqs. (3) and (5) the topological currents that bosonize the fermionic ones, $\bar{\psi}\gamma^\mu\psi \leftrightarrow \epsilon^{\mu\nu}\partial_\nu\phi$ and $\bar{\psi}\gamma^\mu\psi \leftrightarrow \epsilon^{\mu\nu\rho}\partial_\nu A_\rho$, in one and two spatial dimensions, respectively.

At this point, let us include a general current interaction term,

$$Z[s] = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{iK_F[\psi] + iI[j^\mu] - i \int d^\nu x j^\mu s_\mu}, \quad (7)$$

where $I[j^\mu]$ is represented in terms of a functional Fourier transform,

$$\exp\{iI[j^\mu]\} = \mathcal{N} \int \mathcal{D}a_\mu \exp\left\{-i \int d^\nu x h(\mathbf{x}) a_\mu j^\mu + iS[a_\mu]\right\}. \quad (8)$$

The constant \mathcal{N} is chosen such that $I[0] = 0$. The function $h(\mathbf{x})$ is introduced in order to localize the interaction to a spatial region Ω (\mathbf{x} is the spatial part of x). In other words, $h(\mathbf{x})$ is a smooth function which is zero, outside Ω , and it grows to $h(\mathbf{x}) = 1$, inside Ω . By construction, $I[j^\mu] = 0$ for currents localized outside Ω . The role of the smooth function $h(\mathbf{x})$ is to implement the adiabatic contact of the interacting region to the noninteracting one.

Note that for a generic non quadratic interaction $I[j^\mu]$, the computation of $S(a)$ is, in general, not possible. However, inserting Eq. (8) into Eq. (7) and bosonizing the fermions as in a free theory with an external source $s_\mu + h(\mathbf{x})a_\mu$ we find

$$Z[s] = \int \mathcal{D}\phi e^{iK_B[\phi] + iI[\epsilon^{\mu\nu}\partial_\nu\phi] - i \int d^2x s_\mu \epsilon^{\mu\nu\rho} \partial_\nu \phi}, \quad (9)$$

for the 1D case and

$$Z[s] = \int \mathcal{D}A e^{iK_B[A] + iI[\epsilon^{\mu\nu\rho}\partial_\nu A_\rho] - i \int d^3x s_\mu \epsilon^{\mu\nu\rho} \partial_\nu A_\rho}, \quad (10)$$

for the 2D case. That is, we get the universality of the bosonization rules for the currents²⁴.

For $\nu = 2$,

$$K_F[\psi] + I[j^\mu] - \int d^2x s_\mu j^\mu \leftrightarrow K_B[\phi] + I[\epsilon^{\mu\nu} \partial_\nu \phi] - \int d^2x s_\mu \epsilon^{\mu\nu} \partial_\nu \phi. \quad (11)$$

For $\nu = 3$,

$$K_F[\psi] + I[j^\mu] - \int d^3x s_\mu j^\mu \leftrightarrow K_B[A] + I[\epsilon^{\mu\nu\rho} \partial_\nu A_\rho] - \int d^3x s_\mu \epsilon^{\mu\nu\rho} \partial_\nu A_\rho. \quad (12)$$

At this point some comments are in order. Although Eqs. (11) and (12) are similar, the status of bosonization in $(1+1)$ and $(2+1)$ dimensions is different. The bosonized action $K[\phi]$ in Eq. (11) is a local functional of ϕ at low as well as at high energies. This is a consequence of the constraints imposed by the space dimensionality, which are not present in the $(2+1)$ D case. For instance, in a massless $(1+1)$ D fermionic free theory it is possible to build up (zero eigenvalue) bosonic normalizable eigenstates of the $P_\mu P^\mu$ operator. These modes are represented by the topologically trivial sector of the corresponding bosonized theory.

In a $(2+1)$ D fermionic theory it is not possible to construct such normalizable eigenstates for any value of the mass. As a consequence, one has to be careful about the meaning of the bosonizing field A_μ .

In Ref. 28 we have addressed this question in detail showing that the topologically trivial sector of the bosonized action $K_B[A]$ has the vacuum as the only asymptotic state. This can be seen as follows.

If a large mass limit is considered, the bosonized action for free relativistic fermions takes the form of a Chern-Simons term and the next correction is a Maxwell term^{20,21}. Then, we could naively imply the existence of a bosonic mode associated to the Maxwell-Chern-Simons (MCS) theory. Moreover, other corrections would be higher derivative terms implying unphysical modes. In fact, this is not reliable as all these modes would be at a mass scale where the approximation is not valid.

In Ref. 23 we considered a quadratic approximation instead, where the full momentum dependence of the fermionic effective action in Eq. (6) were maintained to obtain a nonlocal

MCS bosonized theory. The Schwinger quantization for these kind of theories, containing nonlocal kinetic terms, were developed in Refs. 29,30. Following these results, we were able to relate²⁸ the mass weight function of the nonlocal MCS bosonized theory and the cross section for fermion pair creation. Then, as there are no bound states, we were able to imply that the mass weight function of the bosonized theory has no delta singularities, i.e., the only asymptotic state in the corresponding topologically trivial sector is the vacuum. Then, while in $(1+1)$ D the bosonizing field ϕ represents some collective excitations of the fermions, in $(2+1)$ D the bosonizing field A_μ does not.

However, when we study 2D (parity breaking) fermionic systems with gapped excitations, the response to a *small* electric field, or an infinitesimal variation of the chemical potential between two regions, will be a quasi-equilibrium property dominated by the low lying energy degrees of freedom. In this regime, the physically relevant quantities are the currents.

In these cases the bosonization technique will implement a sort of hydrodynamical approximation where the conserved currents are represented by $\epsilon^{\mu\nu\rho}\partial_\nu A_\rho$, and the dynamics is given by Eq. (12). In the last sections we will follow this route to implement an alternative calculation for the transverse conductance in 2D fermionic systems.

III. UNIVERSALITY OF LANDAUER CONDUCTANCE AND BOSONIZATION RULES IN 1D SYSTEMS

Let us summarize in this section the results of Ref. 6 where we have shown that the universality of the free bosonization rules implies the universality of Landauer conductance at $T = 0$, for a general class of 1D systems. There, we have considered a Fermi-liquid/finite-system/Fermi-liquid structure where the finite-system presents a quiral symmetry and is adiabatically connected to the reservoirs. For a spinless Fermi-liquid, this amounts to considering for K_F , in Eq. (1), the usual action for massless fermions in one dimension, and a smooth $h(\mathbf{x})$ which is zero outside a finite region extending from $-L/2$ to $+L/2$ (where we

have Fermi-liquid) and it grows to 1 inside this region (interacting region).

Recalling that the free fermionic effective action in Eq. (4) is quadratic, the path integral over b_μ can be simply calculated to obtain the well known bosonized action

$$K_B[\phi] = \int d^2x \frac{\pi}{2} \partial_\mu \phi \partial^\mu \phi,$$

and the bosonized equation of motion corresponding to Eq. (11) can be written as an anomalous (chiral current) divergence,

$$\partial_\mu \left[\partial^\mu \phi + \frac{1}{\pi} \epsilon^{\mu\nu} \frac{\delta I}{\delta j^\nu(x)} \right] = -\frac{1}{\pi} E(\mathbf{x}, t), \quad (13)$$

where $E(\mathbf{x}, t) = \partial_0 s_1(\mathbf{x}, t) - \partial_1 s_0(\mathbf{x}, t)$.

Then, following Maslov and Stone's reasoning², we considered an electric field that is switched on until it saturates in a value $E(\mathbf{x})$. In this case, the large t asymptotic behavior is given by $\phi(\mathbf{x}) = f(\mathbf{x}) - kt$. Replacing this behavior in Eq. (13), we have

$$\partial_x \left[\partial_x f + \frac{1}{\pi} \frac{\delta I}{\delta j^0(x)} \right] = \frac{1}{\pi} E(\mathbf{x}). \quad (14)$$

Due to causality and the fact that, outside the interacting region, Eq. (13) reduces to the free wave equation, we must have $f(\mathbf{x}) = \pm k\mathbf{x} + \phi_0$, the plus (minus) sign corresponding to the right (left) side of this region. Also notice that the contribution to the axial current coming from the interaction is localized. Therefore, the integration of Eq. (14) over the spatial coordinate leads to $\partial_x f(b) - \partial_x f(a) = \frac{1}{\pi} [V(a) - V(b)]$, where a (resp. b) lies on the left (resp. right) side of the finite system, where the electric field is supposed to be zero. This fixes the constant k to be $2k = \frac{1}{\pi} [V(a) - V(b)]$ and the electric current (in bosonized language) results

$$I = \partial_0 \phi = -k = \frac{1}{2\pi} [V(b) - V(a)]. \quad (15)$$

This is the perfect conductance e^2/h in units where $\hbar = e = 1$.

If the mass term $m(x)$ is nonzero (a local gap), chiral symmetry is explicitly broken and the conductance is expected to be suppressed. This happens in the case of a Peierls dielectric

system. The *quantum* regime of a CDW system is described by a complex order parameter $\Delta(x)$, representing the lattice degrees of freedom³¹, whose coupling to the fermions is given by

$$\Delta\bar{\psi}P_L\psi + \bar{\Delta}\bar{\psi}P_R\psi, \quad (16)$$

$P_{R,L} = (1 \pm \gamma_5)/2$ are the projectors corresponding to the right and left modes, respectively. In this case the fermionic effective action is not known, however, the bosonized action is known to be a Sine-Gordon model⁸. For instance, in a path integral framework, this can be obtained⁹, by taking the Lorentz gauge, and using a lagrange multiplier ω to write

$$e^{iK_B(\phi)} = \frac{1}{N} \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}\omega \, e^{i \int d^2x \, \bar{\psi} i \not{\partial} \psi + \Delta \bar{\psi} P_L \psi + \bar{\Delta} \bar{\psi} P_R \psi \delta[\bar{\psi} \gamma^\mu \psi + \epsilon^{\mu\nu} \partial_\nu \phi - \partial^\mu \omega]}. \quad (17)$$

Studying the behavior of the representation (17) under chiral transformations, the well known bosonized action,

$$K_B(\phi) = \frac{\pi}{2} \partial_\mu \phi \partial^\mu \phi + \frac{A}{2} \left(\bar{\Delta} e^{-i\beta\phi} + \Delta e^{i\beta\phi} \right), \quad (18)$$

can be derived (A is a renormalization constant). Since in our case the order parameter Δ has a dynamics given by

$$\mathcal{L}_{\text{ph}}[\Delta] = \frac{1}{2v} \left(\partial_0 \bar{\Delta} \partial_0 \Delta - v^2 \partial_1 \bar{\Delta} \partial_1 \Delta \right) - \frac{\omega_p^2}{2v} \bar{\Delta} \Delta, \quad (19)$$

the chiral symmetry is restored. Then, considering a finite CDW system localized adiabatically ($\Delta(x) \rightarrow h(\mathbf{x})\Delta(x)$) to Fermi-liquid leads, with an additional (local) current interaction of the form shown in Eq. (8), the field equations of motion lead to an anomalous chiral current,

$$\partial_\mu j_A^\mu = -\frac{1}{\pi} E(\mathbf{x}, t), \quad (20)$$

where the total axial current density components are

$$j_0^A = \frac{i\beta h^2}{2v} \left(\Delta^\dagger \partial_0 \Delta \right) - \left(\Delta \partial_0 \Delta^\dagger \right) + \frac{1}{\sqrt{\pi}} \frac{\delta I}{\delta j^1(x)} + \partial_0 \phi \quad (21)$$

and

$$j_1^A = \frac{i\beta v}{2} \left[(h\Delta^\dagger) \partial_1 (h\Delta) - \partial_1 (h\Delta^\dagger) (h\Delta) \right] - \frac{1}{\sqrt{\pi}} \frac{\delta I}{\delta j^0(x)} + \partial_1 \phi. \quad (22)$$

In this equation, the chiral current contains the free fermion contribution modified by terms coming from the lattice degrees of freedom and the current interactions, which are localized in the junction. Here again, the anomalous chiral current divergence leads to a perfect conductance⁶. For instance, these conclusions hold when forward-scattering impurities are present in the CDW junction; if impurities are also present in the Fermi-liquid leads, however, some renormalization of the conductance is expected, in agreement with the results of Ref. 7.

IV. UNIVERSALITY OF TRANSVERSE CONDUCTANCE AND BOSONIZATION RULES IN 2D SYSTEMS

Here, we will show that in the same way that (1+1)D bosonization (cf. Eq. 11) allows a simple calculation of universal perfect Landauer conductance for 1D fermionic systems (cf. Eq. 15), in (2+1)D, the bosonized expressions (12) will allow an exact calculation of universal transport properties in (parity breaking) fermionic systems with gapped excitations. In this section we will use, as an example, the relativistic Dirac field since it is the simplest case that displays the required symmetry properties. In the next section we will extend our results to nonrelativistic systems.

In order to achieve the full bosonization program, we are faced with the problem of computing the bosonized action $K_B(A)$ for free fermions. This amounts to computing the functional transverse Fourier transform of the massive (2+1)D fermionic determinant (Eq. 6). Although a closed expression for $K_B(A)$ is lacking, rather interesting results have been already established. For instance, in the infinite mass limit $m \rightarrow \infty$, the effective action corresponding to the fermionic determinant is given exactly by a local Chern-Simons term, and its transverse Fourier transform can be computed straightforwardly, yielding a Chern-Simons bosonized action²⁰. Another approximation scheme was considered in Ref. 23, where

the full quadratic part of the fermionic effective action, corresponding to the exact expression of the vacuum polarization tensor, has been taken into account. In this case, the approximated bosonized action takes the form of a nonlocal Maxwell-Chern-Simons term. It is worth underlining here that this approximation has been proven to be very useful in order to discuss on an equal footing both the massless and the infinite mass limit corresponding, respectively, to the bosonized actions obtained in Refs. 19 and 20. Also, in Ref. 28, we have seen that this nonlocal bosonized theory is physically well defined, as the associated mass weight function is positive definite, in contrast with the result that would be obtained in any (higher order) derivative approximation (see also the discussion in section II).

Beyond the quadratic approximation for the fermionic determinant, the evaluation of the bosonized action is, in general, a difficult task. Using the results of Ref. 32, some general features of the bosonized action can be obtained. Recently, we have shown²⁴ that the bosonized action $K_B(A)$ in Eq. (12) can be cast in the form of a pure Chern-Simons term, up to a nonlinear and nonlocal redefinition of the gauge field. In this way we can separate the topological information from the particular (and unknown) details of the bosonized action. For our present purposes this representation is not needed. However, we will take advantage of the general structure underlying this separation.

Now, we will present a relationship between the result of Ref. 24, i.e., the universal character of the current bosonization rules for 2D interacting fermionic systems (cf. Eq. (12)), and the universality of transverse conductance between “perfect Hall regions” (where the parity breaking parameter goes to infinity). This is the 2D counterpart of the relationship between universality of current bosonization rules and Landauer conductance, in 1D systems.

At this point one question naturally arises: how can we try to obtain some exact result if we do not even know the exact expression for the bosonized kinetic action $K_B[A]$? The key point is that the unknown terms in the bosonized kinetic action will have the same form of the bosonized interaction term, and when looking at universal behavior, the detailed form

of these terms will be irrelevant. Physically, in analogy with the 1D case, where the Fermi-liquid in the reservoirs imposes strong constraints on the Landauer conductance, we will see that perfect Hall regions will impose strong constraints on the transverse conductance.

Let us consider relativistic 2D fermions with a position dependent (positive definite) mass $m(\mathbf{x})$,

$$K_F = \int d^3x \bar{\psi}(i\not{\partial} + m(\mathbf{x}))\psi. \quad (23)$$

We will suppose that there are at least two disconnected regions or “islands” where the gap $m(\mathbf{x})$ goes to infinity. The multiply connected region around the islands will be called Ω . For definiteness, we will consider a mass parameter which takes a value M ($M \rightarrow \infty$), inside the islands, while it rapidly decreases to a finite value m outside them.

Firstly, since the bosonized action $K_B(A)$ is obtained from a functional transverse Fourier transform of the fermionic determinant (cf. Eq. (6)), it is gauge invariant. Secondly, it is easy to see that in the case of uniform mass, the bosonized kinetic term $K_B(A)$ can be written as (see for example Refs. 24,32)

$$K_{\text{hom}}[A] = \frac{1}{\eta} S_{CS} + \tilde{R}_{\text{hom}}[\epsilon \partial A], \quad (24)$$

where

$$S_{CS} = \frac{1}{2} \int d^3x A_\mu \epsilon^{\mu\nu\rho} \partial_\nu A_\rho, \quad (25)$$

$\eta = \frac{M}{|M|} \frac{1}{4\pi}$ and \tilde{R}_{hom} goes to zero as the mass goes to infinity. A similar conclusion applies to the case where the mass is x -dependent, i.e., when $m(\mathbf{x})$ is replaced by $\lambda m(\mathbf{x})$, and $\lambda \rightarrow \infty$, we have (including a gauge fixing factor in Eq. (6))

$$\lim_{\lambda \rightarrow \infty} \exp iK_B(A) = \int \mathcal{D}b_\mu F(\partial b) e^{i\eta S_{CS}(b) + i \int d^3x \epsilon^{\mu\nu\rho} A_\mu \partial_\nu b_\rho}, \quad (26)$$

where we have used that the large distance behavior of the fermionic effective action is dominated by $\eta S_{CS}(b)$. Integrating over b_μ we get,

$$\lim_{\lambda \rightarrow \infty} K_B(A) = \frac{1}{\eta} S_{CS}(A) . \quad (27)$$

Thus, the bosonized action contains a local Chern-Simons term, corresponding to the bosonization in the infinite mass limit, first obtained in Ref. 20. Therefore, based on gauge invariance of the bosonized action, for any finite value of λ we can write

$$K_B(A) = \frac{1}{\eta} S_{CS}(A) + R[\epsilon \partial A] , \quad (28)$$

where

$$\lim_{\lambda \rightarrow \infty} R[\epsilon \partial A] = 0 . \quad (29)$$

In particular, setting $\lambda = 1$ in Eq. (28), a pure local Chern-Simons term, with parameter $\frac{1}{\eta}$, can be isolated from the bosonized kinetic action for fermions with mass parameter $m(\mathbf{x})$. The remaining part is a gauge invariant functional $R[\epsilon \partial A]$ where every term contains a nontrivial dependence on $m(\mathbf{x})$, which goes to zero when $m(\mathbf{x})$ is replaced by $\lambda m(\mathbf{x})$, and $\lambda \rightarrow \infty$. Then, we expect that when considering any local derivative expansion of R , inside the islands, where the mass parameter takes the value M ($M \rightarrow \infty$), the bosonized kinetic action takes the form of a pure Chern-Simons term with parameter $\frac{1}{\eta}$. In other words, the functional $R[\epsilon \partial A]$ is localized in Ω , i.e., when the support of $\epsilon \partial A$ is localized outside, we have $R[\epsilon \partial A] = 0$. Equivalently, if we write

$$R[\epsilon \partial A] = \int d^3x \mathcal{R}(\epsilon \partial A), \quad (30)$$

the local density \mathcal{R} is zero when evaluating $\epsilon \partial A$ in a point outside Ω .

In order to see this behavior more clearly, let us take the representation (12), replace the expression (23) of the fermionic partition function and integrate over b_μ ,

$$e^{iK_B(A)} = \int \mathcal{D}b_\mu F(\partial b) \int \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{i \int d^3x \bar{\psi}(i\partial + m(x,y) + i\not{b})\psi} e^{i \int d^3x \epsilon^{\mu\nu\rho} A_\mu \partial_\nu b_\rho}. \quad (31)$$

Proceeding in a similar way to Burgess and Quevedo⁹, we can take the Lorentz gauge, and use a lagrange multiplier ω to express

$$e^{iK_B(A)} = \frac{1}{N} \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}\omega \, e^{i \int d^3x \, \bar{\psi}(i\partial + m(\mathbf{x}))\psi} \delta[\bar{\psi}\gamma^\mu\psi + \epsilon^{\mu\nu\rho}\partial_\nu A_\rho - \partial^\mu\omega], \quad (32)$$

where N is chosen such that $K_B(0) = 0$,

$$N = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}\omega \, e^{i \int d^3x \, \bar{\psi}(i\partial + m(x,y))\psi} \delta[\bar{\psi}\gamma^\mu\psi - \partial^\mu\omega]. \quad (33)$$

(Eq. (32) should be compared with its 1D counterpart (17)).

Using a lattice regularization of the path integral in Eq. (32), we see that when $\epsilon\partial A$ is localized outside Ω , the contribution to the nontrivial A dependence comes from those ψ 's which are coupled to $m(\mathbf{x})$ outside Ω . Therefore, for $\epsilon\partial A$ localized on this region (inside the islands), we can compute (32) replacing $m(\mathbf{x})$ by M ($M \rightarrow \infty$), and a Pure Chern-Simons action is obtained. Summarizing, because of locality, the bosonized action is basically a pure Chern-Simons term on the islands, the only possible excitations there corresponding to (nondissipative) currents which are transverse to the external electric field.

Including a current dependent interaction $I[j]$, and using the universality of the bosonization rules for the currents (cf. Eq.(12)), we are left with the complete bosonized action for a 2D (relativistic) fermionic system

$$\frac{1}{\eta} S_{CS}(A) + R[\epsilon\partial A] + I[\epsilon\partial A] + \int d^3x \, s^\mu \epsilon_{\mu\nu\rho} \partial^\nu A^\rho, \quad (34)$$

where s_μ is the external source (we will suppose that the external electric and magnetic fields are localized in Ω). Note that the unknown terms in the bosonized action have the same form of the bosonized interaction term, and they have the same localization properties. As anticipated, this is the reason why we can obtain *exact* results, when looking for universal behaviors.

The corresponding equations of motion are

$$\frac{1}{\eta} \epsilon^{\mu\nu\rho} \partial_\nu A_\rho - \epsilon^{\mu\nu\rho} \partial_\nu \frac{\delta(R + I)}{\delta j^\rho(x)} = -\epsilon^{\mu\nu\rho} \partial_\nu s_\rho. \quad (35)$$

(here, the combination $\epsilon^{\mu\nu\rho} \partial_\nu A_\rho$ has been called j^μ). Taking the i component of Eq. (35), and considering stationary external sources, we are left with the equation

$$\frac{1}{\eta} \partial_k A_0 - \partial_k \frac{\delta(R+I)}{\delta j^0(x)} = -\partial_k s_0, \quad (36)$$

and integrating on a curve that goes from a point \mathbf{a} in the interior of a perfect Hall region (island) to a point \mathbf{b} on another,

$$\frac{1}{\eta} \left(\int d\mathbf{x} \cdot \nabla A_0 \right) - \frac{\delta(R+I)}{\delta j^0(x)} \Big|_{\mathbf{a}}^{\mathbf{b}} = V(\mathbf{b}) - V(\mathbf{a}). \quad (37)$$

The first term corresponds to the bosonized expression of the transverse current,

$$I_t = \int d\mathbf{n} \cdot \mathbf{j}, \quad (38)$$

where $d\mathbf{n}$ is a normal element with respect to the integration curve, whose components are $dn_i = \epsilon_{ij} dx_j$. On the other hand, the last term of the first member is zero as the interaction (I) and the bosonized kinetic part which is not Chern-Simons (R) are localized in Ω . The second member is the electric potential difference between both regions.

Summarizing, our result is that the transverse current I_t , between two “perfect Hall regions” does not depend on the current interactions localized outside them, nor on the particular geometry of these regions, and is given by

$$I_t = \frac{1}{4\pi} (V(b) - V(a)), \quad (39)$$

which corresponds to a transverse conductance $\frac{1}{2} \left(\frac{e^2}{h} \right)$ (in ordinary units). As before, the fundamental role played by the adiabatic transition between the interacting and the non-interacting regions becomes evident. It permits a unified treatment of the perfect Hall and interacting regions in a single theory, establishing a particular matching between them, which corresponds to nondissipative contacts.

The particular value of the conductance $(1/2)e^2/h$ is entirely due to relativistic invariance. Moreover, to the best of our knowledge, the massive Dirac fermion is the only local system with transverse conductance $1/2$, and *finite* (although not universal) longitudinal conductance. For this reason, it could be related to models describing quantum critical properties for transitions between plateaux’s in the Quantum Hall Effect³³.

Now, let us suppose the case where Ω is a circle. Taking $\mu = 0$ in Eq. (35), and integrating over a region S containing this circle,

$$\frac{1}{\eta} \int d^2x \epsilon^{ik} \partial_i A_k - \epsilon^{ik} \partial_i \frac{\delta(R+I)}{\delta j^k(x)} = - \int d^2x \epsilon^{ik} \partial_i s_k. \quad (40)$$

Using Stoke's theorem, we can pass to an integration over the border of S , which is contained outside Ω . There, the local densities \mathcal{R} and \mathcal{I} are zero obtaining

$$\frac{1}{\eta} \int d^2x \epsilon^{ik} \partial_i A_k = - \frac{1}{4\pi} \oint ds \epsilon^{ik} \partial_i s_k. \quad (41)$$

The first member is the bosonized expression for the electric charge contained in S , while the second member is the magnetic flux through S . In the noninteracting case, this corresponds to the Aharonov-Casher result²⁷ for the ground state of a relativistic fermionic system, and comes from the spectral asymmetry associated to fermions in the presence of an external (\mathbf{x} -dependent) magnetic field. With this calculation we are showing that this relationship is an exact and universal result, independent of the current interactions in Ω .

V. NONRELATIVISTIC FERMIONS IN A MAGNETIC FIELD

In the previous section we have called “perfect Hall regions” those regions where the (parity breaking) mass parameter goes to infinity. In these regions, the bosonized action is a pure Chern-Simons term. We have found that the conductance between these regions is $\frac{1}{2} \frac{e^2}{h}$, whatever the form of the current interactions considered. This value of the conductance is a characteristic of relativistic fermions in vacuum. However, in order to make contact with the quantum Hall effect (integer or fractional) we should consider nonrelativistic fermions at finite density submitted to a magnetic field, perpendicular to the plane. Here, in a similar way to the relativistic case, we shall consider arbitrary interactions localized on a region Ω that is adiabatically connected to regions (“islands”) where the system displays an exact integer Landau quantization. To implement this model, let us consider the following action

$$S = \int d^2x dt \psi^*(x) \left\{ -i\partial_t + es_0 + \mu + \frac{1}{2m} \left[-i\vec{\nabla} + e(\vec{d} + \vec{s}) \right]^2 \right\} \psi(x) + I(\psi^*\psi), \quad (42)$$

where $\vec{\nabla} \times \vec{d} = B_{\text{ext}}(\mathbf{x})$, $I(\psi^*\psi)$ is an arbitrary nonrelativistic interaction localized in Ω (possibly nonlocal and non quadratic), and s_μ is a source introduced to prove the system. In Ω , we shall consider a position dependent magnetic field $B_{\text{ext}}(\mathbf{x})$, which changes adiabatically to some constant value on the islands, where the chemical potential is adjusted in order to have the first Landau level completely filled. Note that outside the islands, in general, there is no Landau quantization.

In Ref. 34 we have shown that for the nonrelativistic interactions consider in this model the current bosonization rules,

$$\rho(x) \rightarrow \vec{\nabla} \times \vec{A} \quad (43)$$

$$j_i(x) \rightarrow \epsilon_{ij} E_j(A), \quad (44)$$

are universal, and the bosonized action can be cast in terms of a gauge field A_μ in the following form

$$S_{\text{bos}} = K_B[A] + I(\vec{\nabla} \times \vec{A}), \quad (45)$$

where

$$e^{iK_B(A)} = \int \mathcal{D}b_\mu e^{\text{Tr} \ln \left(-i\partial_t + eb_0 + \mu + \frac{1}{2m} \left[-i\vec{\nabla} + e(\vec{d} + \vec{b}) \right]^2 \right) + i \int d^2x d\tau b_\mu \epsilon_{\mu\nu\rho} \partial_\nu A_\rho}. \quad (46)$$

The nonrelativistic fermionic determinant is a very complicated object and no exact analytic result is known for the general case. In the gaussian approximation, when $B_{\text{ext}} \approx 0$, the spectrum is gapless and there is no signal of topology in the structure of the determinant³⁴. However when B_{ext} is large and varies slowly, the situation is completely different since the Landau quantization opens gaps in the spectrum. In this case, it is possible to make a gradient expansion of the determinant obtaining^{35–37}

$$\begin{aligned} \text{Tr} \ln \left(-i\partial_t + eb_0 + \mu + \frac{1}{2m} \left[-i\vec{\nabla} + e(\vec{d} + \vec{b}) \right]^2 \right) &= -i \int d\mathbf{x} dt \\ &\left\{ \frac{e^2}{4\pi} \gamma b_\mu \epsilon_{\mu\nu\rho} \partial_\nu b_\rho - \frac{e^2}{2\pi m} \left(\frac{\gamma^2}{2} - \gamma \right) (\vec{\nabla} \times \vec{b} + B_{\text{ext}})^2 + \frac{e^2}{2\pi} \gamma b_0 B_{\text{ext}} \right\} + O(|\frac{eB_{\text{ext}}}{m}|^{-1}), \end{aligned} \quad (47)$$

where

$$\gamma = - \sum_{n=0}^{\infty} \Theta \left[\mu + eb_0 - \left(n + \frac{1}{2} \right) \omega_c \right], \quad (48)$$

and $\omega_c = eB_{\text{ext}}/m$.

Due to the presence of the function γ , Eq. (47) is an extremely complicated non quadratic functional of the field. However, in the limit of constant (and large) B_{ext} , we can adjust the chemical potential in such a way that $\gamma = -1$. This procedure is equivalent to projecting the effective action onto the first Landau level. In this approximation, it is simple to integrate the field b_μ (upon gauge fixing) obtaining the bosonized action

$$\lim_{B_{\text{ext}}/m \gg 1} K_B(A) \equiv K_\infty(A) = \left(\frac{2\pi}{e^2} \right) \int d^2x d\tau \left\{ \frac{1}{2} A_\mu \epsilon_{\mu\nu\rho} \partial_\nu A_\rho - \frac{3}{2m} (\vec{\nabla} \times \vec{A})^2 + A_0 B_{\text{ext}} \right\}. \quad (49)$$

Note that this action is not of the pure Chern-Simons form. This is related to the possibility of inducing currents by means of magnetic field inhomogeneities. Also, the last term ($A_0 B_{\text{ext}}$) indicates that we are considering fermions at finite density, where the ground state charge density is proportional to B_{ext} . However, the main point at this moment is that, since the exact bosonized action K_B is gauge invariant (cf. Eq. (46)), it can be cast in the form

$$K_B(A) = K_\infty(A) + R(\vec{j}(A), \rho(A)), \quad (50)$$

where

$$\lim_{eB_{\text{ext}}/m \rightarrow \infty} R(\vec{j}(A), \rho(A)) = 0. \quad (51)$$

(here we have written the gauge invariant variables, in the functional R , in terms of $\rho(A)$ and $\vec{j}(A)$, the bosonized density and currents). It is also important to notice that while Lorentz covariance is lost (since the system is nonrelativistic), the bosonized action remains gauge invariant. Gauge symmetry is precisely one of the ingredients we need to show universal behavior in 2D systems.

Including the nonrelativistic fermionic interactions and using the universality of the bosonization rules for the currents, we can write the complete bosonized action for 2D nonrelativistic fermions as

$$S_{\text{bos}}(A) = K_{\infty}(A) + R[\vec{j}(A), \rho(A)] + I[\rho(A)] + i \int d^3x s^{\mu} \epsilon_{\mu\nu\rho} \partial^{\nu} A^{\rho}. \quad (52)$$

For time-independent external sources, the stationary equations of motion corresponding to the bosonized action (52) read

$$\frac{\delta S_{\text{bos}}}{\delta A_0} = 0 \rightarrow \frac{2\pi}{e^2} \left[\vec{\nabla} \times \vec{A} + B_{\text{ext}} \right] + \vec{\nabla} \times \frac{\delta R}{\delta \vec{j}} = -\vec{\nabla} \times \vec{s}, \quad (53)$$

$$\frac{\delta S_{\text{bos}}}{\delta A_k} = 0 \rightarrow \frac{2\pi}{e^2} \left[\vec{\nabla} A_0 - \frac{3}{m} \vec{\nabla} (\vec{\nabla} \times A) \right] - \vec{\nabla} \left(\frac{\delta(R+I)}{\delta \rho} \right) = \vec{\nabla} s_0. \quad (54)$$

Replacing (53) in (54) we find

$$\frac{2\pi}{e^2} \vec{\nabla} A_0 + \frac{3}{m} \vec{\nabla} \left(\vec{\nabla} \times \frac{\delta R}{\delta \vec{j}} + \frac{2\pi}{e^2} B_{\text{ext}} \right) - \vec{\nabla} \left(\frac{\delta(R+I)}{\delta \rho} \right) = \vec{\nabla} \left(s_0 - \frac{3}{m} \vec{\nabla} \times \vec{s} \right). \quad (55)$$

The last term $(\vec{\nabla} \times \vec{s})$ comes from the fact that in this system it is possible to induce currents by applying an inhomogeneous magnetic field perpendicular to the plane. To calculate the conductance we consider only an external electric field $(\vec{\nabla} \times \vec{s} = 0)$. Then, integrating the last equation along a line with endpoints on different islands, where the Landau quantization is exact, we obtain

$$\frac{2\pi}{e^2} \int d\vec{x} \cdot \vec{\nabla} A_0 + \frac{3}{2m} \left(\vec{\nabla} \times \frac{\delta R}{\delta \vec{j}} + B_{\text{ext}} \right) \Big|_{\mathbf{a}}^{\mathbf{b}} - \frac{\delta(R+I)}{\delta \rho} \Big|_{\mathbf{a}}^{\mathbf{b}} = V(\mathbf{b}) - V(\mathbf{a}). \quad (56)$$

The first term is the bosonic version of the transverse current, the second term is zero since, on the islands, the local density associated to R is zero (R is localized in Ω) and B_{ext} is a constant there. The third term is also zero since the interactions are also localized in Ω . So, turning back to usual units we find

$$I_t = \frac{e^2}{h} \{V(\mathbf{b}) - V(\mathbf{a})\}. \quad (57)$$

This means that the transverse conductance is exact and universal (and of course has the correct coefficient $\frac{e^2}{h}$).

Although in this example we have evaluated the conductance between regions in the integer quantum Hall state, a straightforward generalization to the fractional Quantum Hall effect (Laughlin or Jain states) can be done by using the fermionic action proposed by Ana Lopez and Eduardo Fradkin in Ref. 37. In this case, we have to deal with an extra Chern-Simons gauge field (called statistical field) that essentially works attaching fluxes to the charges, building up in this way the concept of composite fermions³⁸. The main idea is that, in the mean field approximation, the “fictitious magnetic field” produced by the statistical Chern-Simons field spreads out, and combines with the real external field to produce an effective magnetic field given by

$$B_{\text{eff}} = B + \langle \mathcal{B} \rangle = B - 2\pi(2s)\bar{\rho}, \quad (58)$$

where $\langle \mathcal{B} \rangle = -2\pi(2s)\bar{\rho}$ is the mean value of the statistical magnetic field, in the mean field approximation ($\bar{\rho}$ is the mean density and $2s$ counts the number of elementary quantum fluxes attached to each particle).

Then, the system displays an effective integer Landau level quantization with an effective magnetic field B_{eff} given by Eq. (58). Thus, having Landau gaps, it is possible to develop a gradient expansion to evaluate the fermionic determinant. The result is almost the same of Eq. (47), so we can project the system into the “first” Landau level and adjust the chemical potential to have an effective filling factor $\nu_{\text{eff}} = 1$. The main difference with the integer case is that when expressing the filling factor in terms of the original “real” magnetic field, one has $\nu = \frac{1}{2s+1}$, this corresponds to the main sequence of plateaux’s described by the Laughlin wave functions. From this point of view, the mean field composite fermion theory for the fractional quantum Hall effect is essentially the theory of the integer effect with a renormalized magnetic field.

This model for the fractional QHE can be bosonized in exactly the same way as in the integer case. Now, the equation of motion, derived from the bosonized action, leads to the transverse current

$$I_t = \left(\frac{1}{2s+1} \right) \left(\frac{e^2}{h} \right) \{V(\mathbf{b}) - V(\mathbf{a})\}. \quad (59)$$

Thus, with the simple argument of universality of the bosonization rules in 2D and with the assumption that regions with perfect integer or fractional Hall quantization (used to measure the conductance) are adiabatically connected to a non quantized interacting region, we were able to deduce that the transverse conductance is exact and universal.

VI. SUMMARY AND CONCLUSIONS

In this paper we have presented a simple way to study universal transport properties for some bidimensional systems, relying on recent studies on the bosonization program.

In one dimension, bosonization is a natural way to display the anomalous properties of chiral symmetry, the physical principle behind the universal Landauer conductance in Fermi-liquid/finite-system/Fermi-liquid structures. In two dimensions, bosonization is a convenient way to display the parity breaking properties of the system and the underlying gauge symmetry, that is, the physical principle behind the universal quantization of the Hall conductance in the presence of impurities.

Although a complete bosonization in 2D is not yet available, we were able to show exact and universal quantization of the transverse conductance between perfect Hall regions, for a whole class of current interactions. In this regard, we note that the unknown functional R in the bosonized kinetic action (cf. Eqs. (34) and (52)), has the same form of the localized interaction term. Then, when looking at universal behavior, the exact form of this unknown term is irrelevant, all we need is that this term be localized outside the perfect Hall regions.

We can summarize our results by saying that, in the same way that in 1D systems, the Fermi-liquid in the reservoirs imposes strong constraints on Landauer's conductance, in 2D systems, the perfect Hall regions impose strong constraints on the transverse conductance. We have also seen that for this properties be operating, the transition between interacting

and noninteracting regions should be adiabatic, this corresponds to nondissipative contacts. As a by product we also showed that the Aharonov-Casher relation between charge and magnetic flux, originally deduced for free relativistic fermions in a magnetic field²⁷, are universal.

In the case of relativistic fermions, the particular parity breaking properties of the ground state leads to a (half) perfect transverse conductance.

In the nonrelativistic case, we showed the universality of the integer and fractional Hall conductance. This fact clearly shows that relativistic covariance is not important to deduce universal transport. Actually, these properties have to do with general structures, such as universality of the topological bosonized currents, localization and gauge invariance properties of the bosonized action, the presence of parity breaking topological terms and the presence of a gap in the charged degrees of freedom.

Since these results are independent of the shape of the regions used to measure the conductance, we could consider, for instance, two regions in a perfect quantum Hall state, adiabatically connected by a straight line potential barrier, where impurities and phonon interactions are present. This tunnel junctions are in fact experimentally available³⁹. As long as the barrier interaction can be considered adiabatically switched off on the bulk, the Hall conductance between the QHE regions should be exact and universal, since it is dominated by the bulk states.

In Refs. 40–42 we have also obtained this universality using an exact model calculation. In those references, a one dimensional effective field theory was used to evaluate transport properties for a barrier between quantum Hall samples, relating the exact quantization to the chiral properties of the model. In the present work we showed an alternative derivation, using general assumptions and no model calculations, extending these properties to the fractional QHE case, for example.

Finally we would like to point out that, although the bosonization program in higher

dimensions is not fully developed, it is an extremely useful technique to obtain universal properties of strongly correlated fermions in a simple and transparent way.

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